THE USE AND ABUSE OF VERTICAL DEFLECTIONS

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ABSTRACT

This paper reviews the deflection of the vertical and its use and abuse in geodetic surveying. Importantly, the deflection of the vertical in Australia will change by around 6” upon the implementation of the Geocentric Datum of Australia (GDA94). Therefore, for some applications, the deflection of the vertical may no longer simply be neglected in survey computations and adjustments. With the release of AUSGeoid98, absolute deflections of the vertical with respect to the GRS80 ellipsoid are now available for this purpose. The improvements made when using these deflections of the vertical in a terrestrial network adjustment on the GDA94 will be demonstrated for a case study in Western Australia.

INTRODUCTION

Almost all terrestrial survey measurements, with the exception of spatial distances, are made with respect to the Earth’s gravity vector, because a spirit bubble is usually used to align survey instruments. Accordingly, these measurements are nominally oriented with respect to the level (equipotential) surfaces and plumblines of the Earth’s gravity field, which undulate and are not parallel in a purely geometrical sense. This renders them impractical for survey computations and the representation of positions. Therefore, account must be made for the orientation of the survey instruments in the Earth’s gravity field, so that the measurements are of practical use.

Historically, geodesists have introduced a mathematically simpler ellipsoid that is a close fit to the geoid (the level surface that closely coincides with mean sea level) over the region to be surveyed and mapped. As the level surfaces and plumblines are orthogonal by definition, this is equivalent to closely aligning the ellipsoidal normals with the plumblines over the area of interest. This was the case with the Australian National Spheroid (ANS), whose orientation was chosen to give a best fit to the level surfaces and plumblines over Australia (Bomford, 1967). The result is that survey measurements made with respect to the gravity vector in Australia can be assumed to have been oriented with respect to the ANS, thereby simplifying survey reductions and computations on the Australian Geodetic Datum (AGD). For most applications, the separation between the geoid and ANS and the angular differences between the plumbline and the ANS ellipsoidal normal could usually be neglected.

With the adoption of the Geocentric Datum of Australia or GDA94 (eg. Featherstone, 1996), these simplifying assumptions will not necessarily remain valid (Featherstone, 1997). This is because the geocentric GRS80 ellipsoid (Moritz, 1980) used for the GDA94 is a best fit to the level surfaces and plumblines of the Earth’s gravity field on a global scale, and does not provide a best fit over Australia. More importantly, survey observations made with respect to the gravity vector do not change with a change of datum (Heiskanen and Moritz, 1967). Therefore, terrestrial surveys conducted after the adoption of the GDA94 are more likely to require that the separation between the geoid and GRS80 and the angular differences between the plumbline and the GRS80 ellipsoidal normal be taken into account during survey data reduction and adjustment.

This paper reviews some of the various definitions of the deflection of the vertical and illustrates its use and abuse in surveying. Importantly, the change to the geocentric GRS80 ellipsoid from the locally oriented ANS ellipsoid represents a change in concept that has implications on the reduction and adjustment of terrestrial survey data. Since the deflection of the vertical in Australia will change by around 6” upon the implementation of the geocentric GRS80 ellipsoid (Featherstone, 1997), the corrections for its effect can no longer be simply ignored. Fortunately, however, with the release of AUSGeoid98 (Johnston and Featherstone, 1998), a model of the deflections of the vertical with respect to the GRS80 ellipsoid is available, which can be used to apply corrections to survey data. A case study in Western Australia will be used to demonstrate the improvements made when using AUSGeoid98 deflections of the vertical in a terrestrial network adjustment on the GDA94.

THE DEFLECTION OF THE VERTICAL

The deflection of the vertical (**θ**) is the angular difference between the direction of the gravity vector (**g**), or plumbline at a point, and the corresponding ellipsoidal normal through the same point for a particular ellipsoid (Figure 1). Since the plumblines are orthogonal to the level surfaces by definition, the deflection of the vertical also gives a measure of the gradient of the level surfaces (including the geoid) with respect to a particular ellipsoid. Accordingly, the deflection of the vertical is classified as absolute when it refers to a geocentric ellipsoid and relative when it refers to a local ellipsoid. Depending on the choice of ellipsoid, the deflection of the vertical can reach 20” in lowland regions and up to 70” in regions of rugged terrain (Bomford, 1980). In Australia, the largest measured deflection of the vertical with respect to the ANS is around 30” (Fryer, 1971).

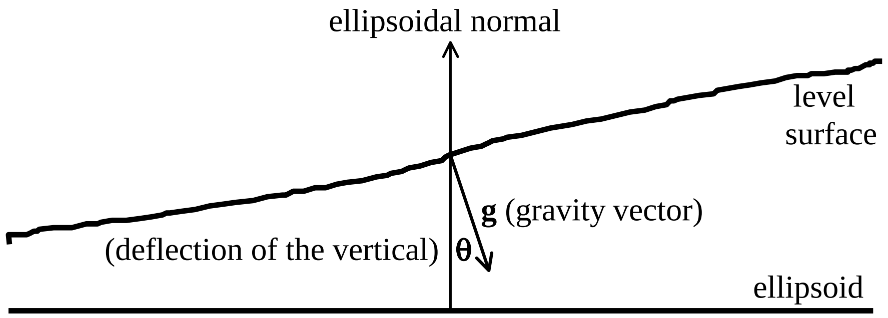

Figure 1. The deflection of the vertical (θ)

The deflection of the vertical, which is a vector quantity, is usually decomposed into two mutually perpendicular components: a north-south or meridional component (ξ), which is reckoned positive northward, and an east-west or prime vertical component (η), which is reckoned positive eastward. In other words, the deflection components are positive if the direction of the gravity vector points further south and further west than the corresponding ellipsoidal normal (Vanicek and Krakiwsky, 1986), or the level surface is rising to the south or west, respectively, with respect to the ellipsoid (Bomford, 1980). These two components reduce to the total deflection of the vertical according to Pythagoras’ theorem

$$\theta^2 = \xi^2 + \eta^2 \quad (1)$$

and the component of the deflection of the vertical can be resolved along a geodetic azimuth (α) by

$$\varepsilon = \xi \cos \alpha + \eta \sin \alpha \quad (2)$$

It is important to distinguish the exact point at which the deflection of the vertical applies, since the deflection of the vertical varies depending upon its position along the plumbline. Equivalently, the direction of the gravity vector varies along the plumbline. This is because the level surfaces and plumblines are curved due to mass distributions inside the Earth's surface. Accordingly, the deflection of the vertical can be defined at the geoid or at any other point, such as at the surface of the Earth. It is acknowledged that there are other subtly different definitions of the deflection of the vertical (Torge, 1991), but only two cases will be considered here.

Vertical Deflection at the Geoid

The deflection of the vertical at the geoid (θ_G) is defined by Pizzetti (Torge, 1991) as the angular difference between the direction of the gravity vector and the ellipsoidal normal through the same point at the geoid. This can be an absolute or relative quantity. However, the vertical deflection at the geoid cannot be directly observed on land because of the presence of the topography. Therefore, deflections of the vertical that are observed at the Earth's surface have to be reduced to the geoid or *vice versa* by accounting for the curvature of the plumbline (described later), which is notoriously problematic.

As an alternative, absolute deflections of the vertical with respect to a geocentric ellipsoid, such as GRS80, can be computed from gravity measurements using Vening-Meinesz's formula (eg. Heiskanen and Moritz, 1967; Vanicek and Krakiwsky, 1986). However, nowadays it is more convenient to relate the deflection of the vertical at the geoid to the gradient of a gravimetric geoid model that has been computed with respect to a geocentric ellipsoid. This can be conceptualised as the reverse process of Helmert astrogeodetic levelling or astrogeodetic geoid determination (eg. Bomford, 1980; Heiskanen and Moritz, 1967). This use of a gravimetric geoid model is considered more convenient because many such models have already been computed for the transformation of GPS ellipsoidal heights to orthometric heights.

The approach is as follows: given a regular grid of gravimetric geoid-ellipsoid separations, the meridional (ξ_G) and prime vertical (η_G) components of the absolute deflection of the vertical at the geoid can be estimated (eg. Torge, 1991) by

$$\xi_G = -\Delta N / (\mu\, \Delta\phi) \quad (3)$$

$$\eta_G = -\Delta N / (\nu\, \Delta\lambda \cos \phi) \quad (4)$$

where the subscript G is used to distinguish these components of the deflection of the vertical at the geoid, μ is the radius of curvature of the GRS80 ellipsoid in the meridian at the point of interest, ν is the radius of curvature of the GRS80 ellipsoid in the prime vertical at the point of interest, ϕ is the geodetic latitude, and ΔN refers to the change in the gravimetric geoid-ellipsoid separation between grid nodes of latitude ($\Delta\phi$) and longitude ($\Delta\lambda$). This approach has been applied to the AUSGeoid98 gravimetric geoid model of Australia (Johnston and Featherstone, 1998) and the east-west and north-south deflection components from GRS80 are available with this product from http://www.auslig.gov.au/geodesy/geoid.htm.

Vertical Deflection at the Earth's Surface

The deflection of the vertical at the surface of the Earth (θ_S) is defined by Helmert (Torge, 1991) as the angular difference between the direction of the gravity vector and the ellipsoidal normal through the same point at the Earth's surface. This can also be an absolute or relative quantity. The deflection of the vertical at the surface of the Earth is of more practical use than the deflection of the vertical at the geoid, because survey measurements are made at the Earth's surface and are thus affected by the deflection of the vertical at this point.

The deflection of the vertical at the Earth's surface can be computed simply by comparing astronomical and geodetic coordinates at the same point on the Earth's surface. The corresponding deflection of the vertical in the prime vertical is the difference between astronomical latitude (Φ) and the geodetic latitude (ϕ) of the same point. Likewise, the deflection of the vertical in the meridian is the difference, scaled for meridional convergence, between astronomical longitude (Λ) and the geodetic longitude (λ) of the same point. These are given, respectively, by

$$\xi_S = \Phi - \phi \tag{5}$$

$$\eta_S = (\Lambda - \lambda) \cos \phi \tag{6}$$

where the subscript S is used to distinguish these components of the deflection of the vertical at the surface of the Earth, and it is assumed that the minor axis of the ellipsoid is parallel to the mean spin axis of the Earth's rotation (Bomford, 1980).

Probably the most important implication of the relations in equations (5) and (6) is to choose the relative deflection of the vertical to be as small as possible through an appropriate orientation of the local ellipsoid. This allows the natural coordinates observed in the Earth's gravity field to be assumed equal to geodetic coordinates on the local ellipsoid. This was the principal reasoning behind the orientation of the ANS in Australia (Bomford, 1967) and why the (now absolute) deflections of the vertical will change by approximately 6" with the use of the geocentric GRS80 ellipsoid (Featherstone, 1997).

Curvature of the Plumbline

As stated, the deflection of the vertical changes with position along the curved plumbline. Therefore, the deflection of the vertical at the geoid (θ_G) does not necessarily equal that at the Earth's surface (θ_S) and *vice versa*. In order to equate these two quantities, the curvature of the plumbline between the geoid and Earth's surface ($\delta\theta_{GS}$) is required. This quantity can not be observed directly because of the presence of the topography, so must be estimated using a model of the Earth's gravity field within the topographic masses.

The curvature of the plumbline can be estimated using an approximate formula (Vanicek and Krakiwsky, 1986; Bomford, 1980), which is based on normal gravity and thus only affects the north-south deflection component ($\delta\xi_{GS}$). This is

$$\delta\xi_{GS} = \delta\theta_{GS} = 0.17'' \sin 2\phi\, H \tag{7}$$

where H is the orthometric height (in kilometres), which is measured along the plumbline between the geoid and surface of the Earth. The evaluation of the actual curvature of the plumbline presents a very difficult task however. This is because exact values of gravity along the plumbline cannot be measured, and for them to be modelled requires detailed knowledge of the mass distribution in the topography. A crude estimate of the curvature of the plumbline is $\delta\varepsilon_{GS}$=3.3" per kilometre in rugged terrain (Vanicek and Krakiwsky, 1986), which makes the typical values in Australia probably less than 1". However, until the actual curvature of the plumbline is known, these approximations are barely useful (Bomford, 1980) and is thus ignored.

THE USE OF VERTICAL DEFLECTIONS

Historically, the most influential use of the deflection of the vertical led to the principle of isostasy, which is used to describe the broad geophysical structure of the Earth's crust. The vertical deflections, observed as part of the 1735-1744 Peruvian expedition to determine whether an oblate or prolate spheroid approximated the figure of the Earth, were shown by Bouguer to be smaller than expected. This and subsequent measurements formed the basis for the two models of isostatic compensation developed by Airy-Heiskanen and Pratt-Heyford. These models are analogous with Archimedes' principle, where the masses of mountains are buoyantly compensated by a thickening of the crust (Airy-Heiskanen model) or a variation in the mass density of the crust (Pratt-Heyford model). However, these two models do not always apply in practice because of the overriding geophysical and mechanical properties of the Earth's crust.

In terrestrial surveying, the deflection of the vertical has three primary uses:

1. transformation of astronomical coordinates to geodetic coordinates;
2. conversion of astronomic azimuth to geodetic azimuth; and
3. reduction of vertical and horizontal angles to the spheroid.

Transformation of Coordinates

The deflections of the vertical provide the transformation between astronomical (natural) coordinates (Φ, Λ), observed with respect to the gravity vector, and the desired geodetic coordinates (ϕ, λ) on the ellipsoid. Rearranging equations (5) and (6), and adhering to the same approximations, gives the coordinate transformation as

$$\phi = \Phi - \xi_S \tag{8}$$

$$\lambda = \Lambda - (\eta_S \sec \phi) \tag{9}$$

where the deflections of the vertical refer to the surface of the Earth, since this is the point at which the astronomic coordinates are usually measured. If the deflections of the vertical at the geoid are used in equations (8) and (9), the limitation imposed by the curvature of the plumbline should be acknowledged.

Laplace's Equation for Azimuth

The deflections of the vertical at the Earth's surface are also required to convert an observed astronomic azimuth (A) to a geodetic azimuth (α). This is achieved using

$$\alpha = A - (\eta_S \tan \phi) - (\xi_S \sin \alpha - \eta_S \cos \alpha) \cot z \qquad (10)$$

where z is the geodetic zenith angle between the observing and observed stations. For most geodetic networks, z is very close to 90 degrees which reduces equation (10) to the well-known Laplace correction

$$\alpha = A - (\eta_S \tan \phi) \qquad (11)$$

The most common use of equation (11) is at Laplace stations, which were used to constrain geodetic azimuth in terrestrial geodetic networks, where systematic atmospheric refraction and undulations in the level surfaces become problematic over longer distances. For example, open-ended traverses were conducted across Australia during the establishment of the AGD. Rather than closing the traverse in a loop, which would increase the survey effort and thus cost, Laplace stations were used to control the azimuth (Bomford, 1967; National Mapping Council, 1986). The Laplace correction must also be applied to stellar observations of astronomic azimuth, if they are to be used to orient a survey network with respect to only a single control point.

Horizontal and Vertical Angles

Horizontal directions and angles have to be corrected for the deflection of the vertical at the Earth's surface when the instrument and target are not coplanar. This can be conceptualised as an error like that encountered due to the misalignment of a theodolite or total station. Assuming that the skew normal correction has been applied between stations (eg. Vanicek and Krakiwsky, 1986), the correction to a measured horizontal direction (Bomford, 1980) is

$$d = D - (\xi_S \sin \alpha - \eta_S \cos \alpha) \tan (90 - z) \qquad (12)$$

where d is the desired direction related to the ellipsoid, D is the measured direction with respect to the gravity vector at the Earth's surface, and $(90-z)$ is the vertical angle between the observing and observed stations. If the observing and observed stations are at the same height above the ellipsoid, then the effect of the deflection of the vertical on horizontal directions is zero.

If this correction is required for horizontal angles instead of directions, the correction term in equation (12) is simply that of the direction with greater azimuth minus that of the direction with smaller azimuth (Bomford, 1980). Alternatively, the corrected directions can be subtracted to give the corrected angles. The error committed due to the neglect of this correction term also propagates along a traverse, hence the need for regular Laplace stations (cf. Bomford, 1967).

Vertical angles also have to be corrected for the deflection of the vertical at the Earth's surface, for exactly the same reasons as horizontal angles. Again, the skew normal corrections and corrections for the Earth's curvature are assumed to have been applied. In the case of a single measured zenith angle, the component of the deflection of the vertical at the Earth's surface in the azimuth of the observation is required. Accordingly, equation (2) for deflections of vertical at the surface of the Earth is applied to the observed zenith angle (Z) to yield the geodetic zenith angle (z) with respect to the ellipsoidal normal (Vanicek and Krakiwsky, 1986)

$$z = Z + (\xi_S \cos \alpha + \eta_S \sin \alpha) \quad (13)$$

In the case of vertical angles, the sign of the correction term is simply reversed. In reciprocal trigonometric levelling, a large proportion of the vertical deflection cancels on differencing. Nevertheless, equation (13) should still be applied to each observation, especially for long baselines.

THE ABUSE OF VERTICAL DEFLECTIONS

The most common abuse of vertical deflections is their neglect in survey reductions and computations. Whilst this was usually acceptable for surveys with respect to the AGD, it will not normally be acceptable for reduction of terrestrial survey data to the GDA94 (Featherstone, 1997). As stated, this is because the generally small relative deflections of the vertical will be replaced by absolute deflections of the vertical, which differ by approximately 6”. Alternatively, surveys on the GDA94 can be designed such that the systematic effects of the deflection of the vertical either cancel or are minimised (cf. Dymock *et al*., 1999). The following examples illustrate the effects of ignoring vertical deflections on measured coordinates, azimuths, and horizontal and vertical angles.

Transformation of Coordinates

Featherstone (1997) has quantified the effect of neglecting the vertical deflection on the transformation of astronomic coordinates to geodetic coordinates on the GDA94. This approach is repeated using the AGD84 to GDA94 transformation parameters (AUSLIG, 1998) and the absolute deflections of the vertical at the geoid computed from AUSGeoid98 (Johnston and Featherstone, 1998). Note that the curvature of the plumbline is neglected in this example, since AUSGeoid98 yields ξ_G and η_G whereas ξ_S and η_S are required in equations (8) and (9). However, given that the deflection of the vertical with respect to the GRS80 ellipsoidal normal is expected to be much larger than the curvature of the plumbline (several seconds versus less than one second), this will have the dominant effect and thus suffice for this comparison.

Table 1 shows GDA94 coordinates of the Johnston origin station, which have been transformed from astronomical coordinates using the AUSGeoid98 deflections of the vertical (equations 8 and 9). The east-west and north-south deflection components have been derived from the AUSGeoid98 grid using bi-cubic interpolation. Table 1 also shows the GDA94 coordinates transformed using the seven-parameter datum transformation (AUSLIG, 1998). The latter assumes a zero deflection of the vertical with respect to the ANS, which is justified for the Johnston station under consideration, since this is the origin point of the AGD (National Mapping Council, 1986).

Astrogeodetic coordinates	*GDA94 geodetic by transformation*

Φ = - 25° 56' 54.55"	ϕ = - 25° 56' 49.34"
Λ = 133° 12' 30.08"	λ = 133° 12' 34.77"
AUSGeoid98 vertical deflections	*GDA94 geodetic by vertical deflection*
ξ_G = + 2.32"	ϕ = - 25° 56' 52.23"
η_G = - 7.93"	λ = 133° 12' 39.62"

Table 1. GDA94 coordinates derived from astrogeodetic coordinates of the Johnston origin using the seven-parameter transformation and the deflections of the vertical.

The results in Table 1 do not support the use of the deflection of the vertical to transform astrogeodetic coordinates, because the derived GDA94 coordinates are not too similar to the seven-parameter-transformed GDA94 coordinates. This is at odds with the analysis conduced by Featherstone (1997) for a systematic 6" change in vertical deflections across Australia. Therefore, the only plausible explanations for this discrepancy are that the AUSGeoid98 deflections of the vertical do not contain sufficient detail to apply coordinate transformations and/or the actual curvature of the plumbline over 571m (the height of the Johnston station) is not negligible.

Laplace's Equation for Azimuth

The Laplace correction (equation 11) to the astronomic azimuth (*A*) introduces a systematic change in the orientation of a survey, for example. This correction can usually be neglected for solar determinations of astronomic azimuth, but not for stellar determinations because of their increased precision. The example shown in Table 2 refers to the GDA94 position of the Johnston origin (Table 1). The east-west deflection of the vertical at the geoid has been determined from AUSGeoid98 using bi-cubic interpolation. Accordingly, the effect of the curvature of the plumbline has also been ignored in this example.

Astrogeodetic azimuth	*Geodetic azimuth*
45° 00' 00.00"	44° 59' 56.14"

Table 2. The effect of the deflection of the vertical on azimuth

From the result in Table 2, the neglect of the Laplace correction causes a change in orientation of a survey by 3.86". For instance, if a radiation is made using the astronomic azimuth instead of the geodetic azimuth over a line of 2km in length, there will be an error of approximately 37mm. However, this example only applies to a survey that relies on an azimuth for its orientation. If two or more known GDA94 coordinates are used as control, these provide the geodetic azimuth, so there is no need to apply Laplace corrections in this instance.

Horizontal and Vertical Angles

In order to illustrate the effect of neglecting deflections of the vertical on measured horizontal directions (*D*) and measured zenith angles (*Z*), the AUSGeoid98 vertical deflections are used in equations (12) and (13), respectively. Again, the curvature of the plumbline is neglected and AUSGeoid98 deflections of the vertical at the geoid are calculated using bi-cubic interpolation. Only the correction terms for the GDA94 position given in Table 1 are computed, since these are independent of the measurement to which the correction applies. In each case, the azimuth is taken as 45 degrees for convenience. The results are summarised in Table 3.

Horizontal direction	*Zenith angle*
00.63” for z = 85°	-03.97”
07.25” for z = 45°	--

Table 3. The effect of the deflections of the vertical on horizontal directions and zenith angles

The example in Table 3 shows that the effect of the deflection of the vertical on horizontal directions and zenith angles can be relatively large. However, it should be pointed out that when these observations are made in conjunction with other observations in the gravity field, a large amount of the effect cancels. Nevertheless, since the deflections of the vertical at the geoid are readily available from AUSGeoid98 and these terms can be computed relatively easily, they should be included so as to reduce their small, yet systematic, effects on the survey results.

A CASE STUDY IN GUILDERTON, WESTERN AUSTRALIA

The following case study applies deflections of the vertical at the geoid, bi-cubically interpolated from AUSGeoid98, to the reduction and adjustment of geodetic survey data collected by final-year surveying students in the School of Spatial Sciences. These data have been collected as part of the units Applied Field Surveying 482 and Applied Geodetic Surveying 482, which are conducted over a 5km by 5km area near Guilderton, Western Australia. This presents a challenging case study because Guilderton is close to the Darling Fault, which is known to cause a large disturbance to the plumblines and level surfaces (eg. Friedlieb *et al*., 1997).

The survey data were corrected for instrument calibrations and atmospheric refraction, then input to *Geolab* version 2.4d (Bitwise Ideas Inc., 1993) for reduction and least-squares adjustment. The data used comprise spatial distances, horizontal directions, vertical angles and an astronomic azimuth with respect to a single control station known on the GDA94. The *Geolab* software corrects for the deflection of the vertical, provided that this information is supplied. Therefore, the deflections of the vertical at the geoid from the AUSGeoid98 model were both omitted and included to study their effect on the network adjustment.

To determine the effectiveness of each approach, the three quality control indicators from a network adjustment (eg. Featherstone *et al*., 1998) were used. A network adjustment is considered successful if:

1. the *a posteriori* variance (sigma-zero) close to unity;
2. the chi-squared hypothesis test on this estimated variance passes; and
3. no outlying measurements remain after the adjustment.

For each network adjustment, the same weights were applied to account for the different observation types, so that the only changes in these three quality control indicators were due to the inclusion of the vertical deflections. Table 4 shows the values of the quality control indicators for the network without and with the vertical deflections at the geoid applied.

Vertical deflections	*A posteriori variance*	*Result of chi-squared test*	*Number of outliers*
Not applied	1.9883	FAIL	2 of 80
Applied	1.1484	PASS	0 of 80

Table 4. The effect of the deflection of the vertical on the three indicators of a successful network adjustment (datum: GDA94)

From the results in Table 4, it is clear that the inclusion of vertical deflections improves the quality of the network adjustment, as indicated by the three standard indicators. However, it is important to acknowledge that there are several other factors that could also contribute to this result, principally the accuracy of the measurements. Nevertheless, given that rigorous and correct geodetic theory is being used, it is more likely that the inclusion of the vertical deflection, albeit at the geoid, provides the most plausible explanation for the improvement.

CONCLUDING REMARK

This paper has reviewed the definition and use of the deflection of the vertical and showed its common uses and abuses in terrestrial surveying. The need to seriously consider the effects of the deflection of the vertical has come about because of the introduction of the GDA94. Since the GRS80 ellipsoid associated with this new datum is not a best fit to the level surfaces and plumblines of the Earth's gravity field over Australia, the associated (absolute) deflections of the vertical generally become larger. Fortunately, however, absolute deflections of the vertical at the geoid are now available for the whole continent as part of the AUSGeoid98 product. Therefore, since this information is available and is consistent with rigorous geodetic theory, it is appropriate to routinely apply corrections for deflections of the vertical to terrestrial survey data.

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